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Relationship between the atomic inversion and Wigner function for multimode multiphoton Jaynes–Cummings model

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Abstract

In this paper we consider the multimode multiphoton Jaynes–Cummings model, which consists of a two-level atom, initially prepared in an excited atomic state, interacting with N modes of electromagnetic field prepared in general pure quantum states. For this system we show that under certain conditions the evolution of the Wigner function at the phase-space origin provides direct information on the corresponding atomic inversion. This relation is also valid even if the system includes Kerr-like nonlinearity, Stark shift effect, different types of initial atomic states as well as moving atoms. Furthermore, based on this fact we discuss for the single-mode case the possibility of detecting the atomic inversion by means of techniques similar to those used for the Wigner function.

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1. Introduction

The Jaynes–Cummings model (JCM) [1] has continued to be the subject of not only theoretical studies but also experimental investigation (see, e.g., [2]). This model in the simplest form is described as a two-level atomic system interacting with an electromagnetic field (for a review see, e.g., [3]). Many of the quantum features of the JCM have been predicted and observed: among the most well known is the revival–collapse phenomenon (RCP) of the atomic inversion $\langle\hat{\sigma}_z(T)\rangle$ [4, 5]. RCP arises from the presence of multiple exchange of photons between the radiating atom and the cavity mode. Observation of RCP for $\langle\hat{\sigma}_z(T)\rangle$ has been performed using the one-atom maser [2], which is more sophisticated than the dynamics of the JCM. In the same respect it has been shown that the measured probability of atomic inversion for specific interaction times turns out to be the symmetrically ordered characteristic function [6]. This scheme is closely related to the nonlinear atomic homodyne detection [7] in which an atom is coupled to two modes of the field, one acting as the signal mode and the other as the local oscillator mode. Furthermore, in [8] a simple scheme, which can slow down considerably the usual exponential decay of the upper state population in an atomic two-level system based on an additional intense field with frequency lower than the total decay width of the atom, is given.

The multiphoton single-mode JCM has been the focus of considerable interest in the literature, e.g. [9–12]. For instance, for this model Heisenberg's equations of motion for the atomic energy operator have been exactly solved [9]. Its phase variance can exhibit RCP about the long-time behaviour [10]. Moreover, the investigation of this model against squeezed light has shown that the atomic inversion can display RCP for general squeezed input but not for squeezed vacuum [11]. The analysis of the model against superposition of squeezed displaced number states, i.e. the most general case, is given in [12]. The multimode version of the JCM has been investigated, in particular, the two-mode JCM, e.g. [13, 14]. The most important result related to the two-mode JCM is that the atomic inversion exhibits the revival–collapse pattern as well as secondary revivals, which are independent of the intensities of the initial modes [14]. Moreover, we can mention [15] in which the Hamiltonian for the multiphoton multimode JCM has been derived from first principles. Nevertheless, the generalizations of the JCM as a nonlinear version in both bosonic and fermonic variables are given in [16], where the exact wavefunction and energy levels are calculated.

Quasiprobability distribution functions are very useful tools in quantum mechanics since they can be used in the calculation of the correlation function of operators as classical-like integrals and in the transition to classical physics. There are three types of such functions, namely, the Wigner W, the Husimi Q and the Glauber P functions [17]. These functions are not real probability functions owing to the position–momentum uncertainty principle. Actually, the W function plays an exceptional role among all quasiprobability distributions for several reasons. It contains complete information about the state of the system (i.e. it carries the same information as the density operator or as the corresponding wavefunction). It provides proper marginal distributions for individual phase-space variables. It can be used to evaluate the symmetrically ordered moments for the operators of the system. It is sensitive to the interference in phase space and consequently it provides a clear prediction of the possible occurrence of the nonclassical effects of the quantum mechanical system. In this respect, the W function can be used to analyse the decoherence of the quantum system, i.e. the process that limits the appearance of quantum effects and turns them into classical phenomena [18, 19]. It is worth mentioning that the decoherence is useful for applications which require keeping coherence in mesoscopic or macroscopic systems such as quantum computation [20]. Finally, the W function can be determined from the knowledge of the complete set of moments of system operators [21].

For the single-mode JCM with field prepared initially in coherent light it has been shown that there is a relation between the behaviour of the Q distribution function and the occurrence of the RCP in $\langle\hat{\sigma}_z(T)\rangle$ [22–25]. For instance, the collapse of the Rabi oscillations in the evolution of $\langle\hat{\sigma}_z(T)\rangle$ is reflected in the behaviour of the Q function as the splitting of the initial shifted Gaussian distribution into two distributions, which counter-rotate on a circle in the complex plane of the distribution. However, the revivals in $\langle\hat{\sigma}_z(T)\rangle$ correspond to the collision of the two peaks of the Q function to produce a single-peak distribution, which is similar to the initial one. It is worthwhile mentioning that such a relation between the Q function and $\langle\hat{\sigma}_z(T)\rangle$ for JCM is remarkable only when the amplitude of the initial coherent light is very large. Additionally, the comparison between $\langle\hat{\sigma}_z(T)\rangle$ and the Q function has to be performed at the same specific values of the interaction time. In this paper we give a new relation, which shows that the information stored in $\langle\hat{\sigma}_z(T)\rangle$ can be obtained from the evolution of the W function at the phase-space origin (WOP). This relation depends on the type of the initial state of the optical cavity field, the values of the transition parameters and the number of modes interacting with the two-level atom. The motivation of the work is two-fold:

(i) $\langle\hat{\sigma}_z(T)\rangle$ can be measured using techniques similar to those used for the W function.
(ii) $\langle\hat{\sigma}_z(T)\rangle$ can be used to provide information on the nonclassicality of the bosonic system.

Actually, these are novel results.

In this paper we consider the interaction of multiphoton N modes of the electromagnetic field with a two-level atom in terms of the multimode multiphoton Jaynes–Cummings model (JCM). The Hamiltonian controlling the system is given in the framework of the rotating wave approximation. We also consider the optical cavity modes which are initially prepared in general pure quantum states. For this system we seek the relation between the evolution of $\langle\hat{\sigma}_z(T)\rangle$ and WOP. This will be done in the following order: in section 2 we give the basic relations and equations used throughout the paper. In section 3 we discuss the main results as well as shed light on how one can measure $\langle\hat{\sigma}_z(T)\rangle$ using techniques similar to those used for the W function. Conclusions and remarks are summarized in section 4.

2. Basic relations and equations

In this section we give the basic relations and equations, which enable us to justify the relationship between $\langle\hat{\sigma}_z(T)\rangle$ and the W function for JCM. Firstly, within the dipole and rotating wave approximation (RWA), the general form of an idealized Hamiltonian, which describes the interaction of a multiphoton N mode cavity field with a two-level atom (JCM) is [15, 26]

$$\frac{\hat{H}}{\hbar}=\sum_{j=1}^{N}\omega_j\hat{a}_j^{\dagger}\hat{a}_j+\omega_a\hat{\sigma}_z+\lambda\left(\hat{\sigma}_+\prod_{j=1}^{N}\hat{a}_j^{k_j}+\hat{\sigma}_-\prod_{j=1}^{N}\hat{a}_j^{\dagger k_j}\right) \tag{1}$$

where the jth mode is designated by $\hat{a}_j$ $\left(\hat{a}_j^{\dagger}\right)$, the usual photon annihilation (creation) operator, the frequency ω_j and the transition parameter k_j. $\hat{\sigma}_{\pm}$ and $\hat{\sigma}_z$ are the Pauli spin operators describing the atomic system, ω_a is the atomic transition frequency and λ is the atom–field coupling constant. The Hamiltonian can be written as the sum of the two operators:

$$\begin{aligned}\hat{C}_1&=\epsilon_1\hat{\sigma}_z+\sum_{j=1}^{N}\omega_j\hat{a}_j^{\dagger}\hat{a}_j\\ \hat{C}_2&=\triangle\hat{\sigma}_z+\lambda\left(\hat{\sigma}_+\prod_{j=1}^{N}\hat{a}_j^{k_j}+\hat{\sigma}_-\prod_{j=1}^{N}\hat{a}_j^{\dagger k_j}\right)\end{aligned} \tag{2}$$

where

$$\epsilon_1=\sum_{j=1}^{N}k_j\omega_j \qquad \triangle=\omega_a-\epsilon_1 \tag{3}$$

and $\triangle$ is the detuning parameter. Based on the standard commutation rules for the bosonic and Pauli operators, it is easy to prove that $\hat{C}_1$ and $\hat{C}_2$ are constants of motion and also that they commute with each other. In the interaction picture the unitary evolution operator takes the form

$$\begin{aligned}
\hat{U}_I(T,0) &= \exp\left(-\mathrm{i}\frac{T}{\lambda}\hat{C}_2\right) = \sum_{n=0}^{\infty}\frac{\left(-\mathrm{i}\frac{T}{\lambda}\hat{C}_2\right)^n}{n!}\\
&= \sum_{n=0}^{\infty}\frac{\left(-\mathrm{i}\frac{T}{\lambda}\hat{C}_2\right)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{\left(-\mathrm{i}\frac{T}{\lambda}\hat{C}_2\right)^{2n+1}}{(2n+1)!}\\
&= \sum_{n=0}^{\infty}\frac{(-1)^n(T\hat{\nu})^{2n}}{(2n)!} - \frac{\mathrm{i}}{\lambda\hat{\nu}}\sum_{n=0}^{\infty}\frac{(-1)^n(T\hat{\nu})^{2n+1}}{(2n+1)!}\hat{C}_2\\
&= \cos(T\hat{\nu}) - \mathrm{i}\frac{\sin(T\hat{\nu})}{\lambda\hat{\nu}}\hat{C}_2
\end{aligned} \tag{4}$$

where

$$T = \lambda t \qquad \hat{\nu}^2 = \left(\frac{\triangle}{\lambda}\right)^2 + \hat{\sigma}_-\hat{\sigma}_+\prod_{j=1}^{N}\hat{a}_j^{\dagger k_j}\hat{a}_j^{k_j} + \hat{\sigma}_+\hat{\sigma}_-\prod_{j=1}^{N}\hat{a}_j^{k_j}\hat{a}_j^{\dagger k_j}. \tag{5}$$

To make the analysis quite general, we assume that the jth mode is initially prepared in a general pure quantum state given by

$$|\psi_j(0)\rangle = \sum_{n_j=0}^{\infty} C_{n_j}^{(j)}|n_j\rangle \tag{6}$$

where $C_{n_j}^{(j)}$ represents the probability amplitude for the state under consideration such that $\sum_{n_j=0}^{\infty}\left|C_{n_j}^{(j)}\right|^2 = 1$. We suppose that the atom is initially prepared in the excited atomic state $|+\rangle$. Therefore, the initial state of the atom–field system can be expressed as

$$\begin{aligned}
|\Psi(0)\rangle &= |\psi_1(0)\rangle\otimes|\psi_2(0)\rangle\otimes\cdots\otimes|\psi_N(0)\rangle\otimes|+\rangle\\
&= \sum_{\underline{n}=\underline{0}}^{\infty} F(n_1,n_2,\ldots,n_N)|+,n_1,n_2,\ldots,n_N\rangle
\end{aligned} \tag{7}$$

where the vector notation in the index means that we have N summations, i.e. $\underline{n} \equiv (n_1,n_2,\ldots,n_N)$, and the distribution $F(n_1,n_2,\ldots,n_N)$ reads

$$F(n_1,n_2,\ldots,n_N) = \prod_{j=1}^{N} C_{n_j}^{(j)}. \tag{8}$$

From (4) and (7) one can easily obtain the dynamical wavefunction for the system in the interaction picture as

$$\begin{aligned}
|\Psi(T)\rangle &= \hat{U}_I(T,0)|\Psi(0)\rangle\\
&= \sum_{\underline{n}=\underline{0}}^{\infty} F(n_1,n_2,\ldots,n_N)[G_1(n_1,n_2,\ldots,n_N,T)|+,n_1,n_2,\ldots,n_N\rangle\\
&\quad - \mathrm{i}G_2(n_1,n_2,\ldots,n_N,T)|-,n_1+k_1,\ldots,n_N+k_N\rangle],
\end{aligned} \tag{9}$$

where

$$h(n_1,n_2,\ldots,n_N;k_1,k_2,\ldots,k_N) = \prod_{j=1}^{N}\frac{(n_j+k_j)!}{n_j!} + \left(\frac{\triangle}{\lambda}\right)^2$$

$$G_1(n_1, n_2, \ldots, n_N, T) = \cos(T\sqrt{h(n_1, n_2, \ldots, n_N; k_1, k_2, \ldots, k_N)})$$
$$- \mathrm{i}\frac{\triangle}{\lambda}\frac{\sin(T\sqrt{h(n_1, n_2, \ldots, n_N; k_1, k_2, \ldots, k_N)})}{\sqrt{h(n_1, n_2, \ldots, n_N; k_1, k_2, \ldots, k_N)}}$$
$$G_2(n_1, n_2, \ldots, n_N, T) = -\frac{\sin(T\sqrt{h(n_1, n_2, \ldots, n_N; k_1, k_2, \ldots, k_N)})}{\sqrt{h(n_1, n_2, \ldots, n_N; k_1, k_2, \ldots, k_N)}}\sqrt{\prod_{j=1}^{N}\frac{(n_j + k_j)!}{n_j!}}. \tag{10}$$

The atomic inversion associated with (9) is

$$\langle\sigma_z(T)\rangle = \sum_{\underline{n}=\underline{0}}^{\infty}|F(n_1, n_2, \ldots, n_N)|^2[|G_1(n_1, n_2, \ldots, n_N, T)|^2 - |G_2(n_1, n_2, \ldots, n_N, T)|^2]. \tag{11}$$

For reasons that will be clear shortly we write down the different forms for the W function. The basis of the W function for any quantum mechanical system is the W function of the number state $|n\rangle$ having the form

$$W_n(q, p) = \frac{(-1)^n}{\pi}\exp(-q^2 - p^2)\mathrm{L}_n(2q^2 + 2p^2) \tag{12}$$

where $\mathrm{L}_n(\cdot)$ is the Laguerre polynomial of order n. Also the marginal position probability distribution for the number state $|n\rangle$ can be obtained from (12) as

$$P(q) = \int_{-\infty}^{\infty} W_n(q, p)\,\mathrm{d}p = \frac{1}{\sqrt{\pi}}\frac{\mathrm{H}_n^2(q)}{2^n n!}\exp\left(-\frac{q^2}{2}\right) \tag{13}$$

where $\mathrm{H}_n(\cdot)$ is the Hermite polynomial of order n. The corresponding form of the marginal momentum probability distribution is the same as (13) but q should be replaced by p. The N-mode dynamical W function can be defined up to a constant prefactor as [17]

$$W(\underline{\beta}, T) = \mathrm{Tr}\left[\hat{\rho}(T)\hat{D}(\underline{\beta})\exp\left(\mathrm{i}\pi\sum_{j=1}^{N}\hat{a}_j^{\dagger}\hat{a}_j\right)\hat{D}^{-1}(\underline{\beta})\right] \tag{14}$$

where $\underline{\beta} = (\beta_1, \beta_2, \ldots, \beta_N) = (q_1, q_2, \ldots, q_N; p_1, p_2, \ldots, p_N)$ since $\beta_j = q_j + \mathrm{i}p_j$. $\hat{\rho}(T)$ is the density matrix for the system under consideration and $\hat{D}(\underline{\beta})$ is the multimode displacement operator having the form

$$\hat{D}(\underline{\beta}) = \exp\left[\sum_{j=1}^{N}\left(\hat{a}_j^{\dagger}\beta_j - \hat{a}_j\beta_j^{*}\right)\right]. \tag{15}$$

At the phase-space origin (i.e. $\underline{\beta} = \underline{0}$) formula (14) reduces to

$$W(\underline{0}, T) = \mathrm{Tr}\left[\hat{\rho}(T)\exp\left(\mathrm{i}\pi\sum_{j=1}^{N}\hat{a}_j^{\dagger}\hat{a}_j\right)\right]. \tag{16}$$

Formula (16) indicates that the main contribution for the W function at the phase-space origin results from the diagonal part of the density matrix of the quantum mechanical system. Comparing this situation with that of atomic inversion one can conclude that there is a clear relationship between the evolution of WOP and the corresponding atomic inversion.

Now on substituting (9) into (16) and carrying out the expectation value we arrive at

$$W(\underline{0},T)=\sum_{\underline{n}=\underline{0}}^{\infty}|F(n_1,n_2,\ldots,n_N)|^2(-1)^{n_1+n_2+\cdots+n_N}\{|G_1(n_1,n_2,\ldots,n_N,T)|^2 + (-1)^{k_1+\cdots+k_N}|G_2(n_1,n_2,\ldots,n_N,T)|^2\}. \tag{17}$$

Expression (17) and its consequences are the main results of the paper. Specifically, we show that for particular types of initial states and particular values of the transition parameters k_j, expression (17) coincides with that of the corresponding atomic inversion. This will be discussed in the following section.

We proceed by connecting the present results with those of the homodyne tomography technique. Therefore, we give the mathematical relation between the N-mode W function and the corresponding distribution function $pr(q_1,\ldots,q_N,\theta_1,\ldots,\theta_N,T)$ (i.e. Radon transformation). Such a relation is just the generalization of the single-mode case and can be expressed as

$$\begin{aligned}pr(q_1,\ldots,q_N,\theta_1,\ldots,\theta_N,T)={}&\int_{-\infty}^{\infty}\mathrm{d}p_1\cdots\int_{-\infty}^{\infty}\mathrm{d}p_N\\&\times W(q_1\cos\theta_1-p_1\sin\theta_1,q_1\sin\theta_1+p_1\cos\theta_1,\ldots,q_N\cos\theta_N\\&-p_N\sin\theta_N,q_N\sin\theta_N+p_N\cos\theta_N,T).\end{aligned} \tag{18}$$

In (18) we have assumed that N modes can be delivered to N separate ideal balance homodyne detectors. At the phase-space origin, i.e. $q_j=0,\theta_j=0$, $j=1,\ldots N$, formula (18) becomes phase independent and reduces to

$$pr(\underline{0},T)=\int_{-\infty}^{\infty}\mathrm{d}p_1\cdots\int_{-\infty}^{\infty}\mathrm{d}p_N\,W(0,p_1,0,p_2,\ldots,0,p_N,T) \tag{19}$$

where $W(0,p_1,0,p_2,\ldots,0,p_N,T)$ is the diagonal part of the W function, which is phase independent. On using (12) and (13) one can easily deduce $pr(\underline{0},T)$ $(=P(\underline{0},T))$ given by (19) for the state vector (9) as

$$P(\underline{0},T)=\sum_{\underline{n}=\underline{0}}^{\infty}|F(n_1,n_2,\ldots,n_N)|^2\left\{|G_1(n_1,n_2,\ldots,n_N,T)|^2\prod_{j=1}^{N}\frac{\mathrm{H}^2_{n_j}(0)}{2^{n_j}n_j!} + |G_2(n_1,n_2,\ldots,n_N,T)|^2\prod_{j=1}^{N}\frac{\mathrm{H}^2_{n_j+k_j}(0)}{2^{n_j+k_j}(n_j+k_j)!}\right\}. \tag{20}$$

3. Main results

In this section we discuss two issues: (i) we investigate the results by making a comparative study among the behaviour of $\langle\hat{\sigma}_z(T)\rangle$, $W(\underline{0},T)$ and $P(\underline{0},T)$, for the system under consideration. (ii) We argue how one can measure the atomic inversion via techniques similar to those used for the W function.

3.1. Investigation of the results

As mentioned above we investigate the behaviour of the quantities $W(\underline{0},T)$ and $P(\underline{0},T)$, and then compare such behaviour with that of the corresponding $\langle\hat{\sigma}_z(T)\rangle$.

We start the discussion with $W(\underline{0},T)$, which is given by (17). It is obvious that when $n_1+n_2+\cdots+n_N$ is even and $k_1+k_2+\cdots+k_N$ is odd, $W(\underline{0},T)$ is identical with the atomic inversion of the system (cf (11)). In other words, the atomic inversion can be used to provide information on the nonclassicality of the dynamical bosonic system. For instance, when the evolution of $\langle\hat{\sigma}_z(T)\rangle$ displays negative values, the JCM can exhibit nonclassical effects. Nevertheless, when $k_1 + k_2 + \cdots + k_N$ is an even number regardless of the value of $n_1 + n_2 + \cdots + n_N$, expression (17) becomes time independent (i.e. $W(\underline{0}, T)$ is localized) and can be factorized in the following sense,

$$W(\underline{0}, T) = \prod_{j=1}^{N} W_j(0, 0) \tag{21}$$

where $W_j(0, 0)$ is the initial value of the W function of the jth mode at the phase-space origin having the form

$$W_j(0, 0) = \sum_{n_j=0}^{\infty} (-1)^{n_j} \left|C_{n_j}^{(j)}\right|^2. \tag{22}$$

Expression (21) can be obtained, e.g., when the number of modes N is even and the transition parameters are symmetric, i.e. $k_1 = k_2 = \cdots = k_N$. Moreover, expression (21) indicates that if the initial W function of only one of the modes has a negative value at the phase-space origin whereas those of the others are positive, the system can provide nonclassical effects. This is a sufficient but not necessary condition.

Now we have a closer look at the behaviour of the $W(0, T)$ for the single-mode case, i.e. $N = 1$. In this case expression (17) reduces to

$$W(0, T) = \sum_{n_1=0}^{\infty} \left|C_{n_1}^{(1)}\right|^2 (-1)^{n_1} \{|G_1(n_1, T)|^2 + (-1)^{k_1} |G_2(n_1, T)|^2\}. \tag{23}$$

For odd transition parameter and initial even (odd) parity states, e.g. even (odd) coherent states, (23) gives

$$W_{\pm}(0, T) = \pm\langle\sigma_z(T)\rangle \tag{24}$$

where '+' and '−' signs denote even and odd parity states, respectively. For $\triangle = 0$, expression (24) indicates that when the initial intensity of the radiation field is weak the system can exhibit nonclassical effects periodically. Nevertheless, in the strong intensity regime one has $W(0, T) \simeq 0\,(\neq 0)$, which is associated with the occurrence of the collapse (revival) in $\langle\hat{\sigma}_z(T)\rangle$. Consequently the W function exhibits nonclassical interference at the phase-space origin only in the course of the revival times, i.e. the nonclassical effects most probably occur in the course of the revival time. However for the non-parity states the locations (in the interaction time domain) of collapses and revivals occurring in $W(0, T)$ are interchanged compared to those in $\langle\hat{\sigma}_z(T)\rangle$. This agrees with the fact that the JCM generates Schrödinger-cat states in the course of the collapse time [12]. We proceed by investigating the behaviour of the $W(0, T)$ for the standard JCM, i.e. $k_1 = 1$, $\triangle = 0$ and the field is initially prepared in coherent light with amplitude $|\alpha|$. In this case (23) reduces to

$$\begin{aligned} W(0, T) &= \exp(-|\alpha|^2) \sum_{n_1=0}^{\infty} \frac{|\alpha|^{2n_1}}{n_1!} (-1)^{n_1} \cos(2T\sqrt{n_1 + 1}) \\ &= \exp(-|\alpha|^2) \sum_{n_1=0}^{\infty} \frac{|\alpha|^{2n_1}}{n_1!} \cos(2T\sqrt{n_1 + 1} + n_1\pi). \end{aligned} \tag{25}$$

Expression (25) is identical with that of the corresponding atomic inversion but with an additional factor, which is $(-1)^{n_1}$. This factor is responsible for the interchange of the

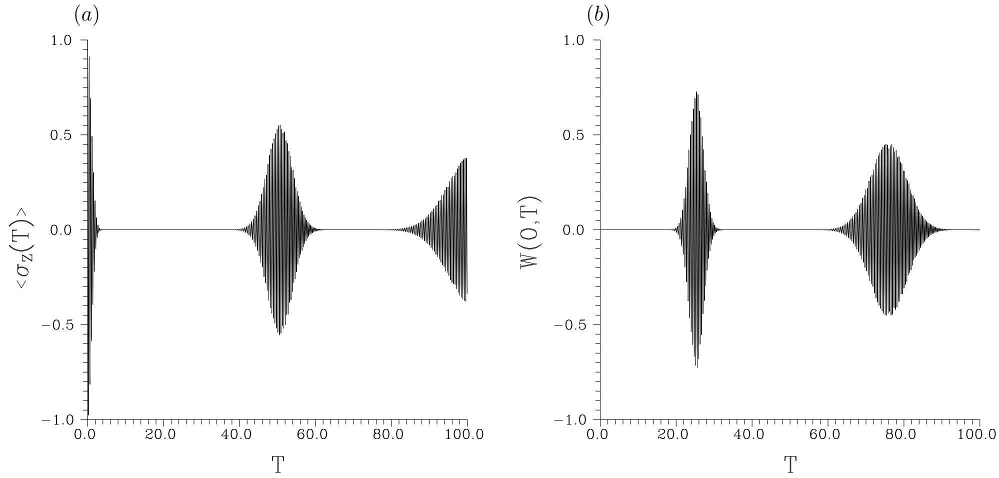

Figure 1. The evolution of the atomic inversion $\langle\hat{\sigma}_z(T)\rangle$ (*a*) and the W function $W(0, T)$ (*b*) against the scaled time T for the single-mode case with $k_1 = 1$ and when the field and atom are initially prepared in the coherent state (with $|\alpha| = 8$) and atomic excited state, respectively.

'locations' of collapses and revivals occurring in $W(0, T)$ compared to those exhibited in $\langle\hat{\sigma}_z(T)\rangle$, as we mentioned above. This is remarkable in figures 1(*a*) and (*b*) where we have plotted $\langle\hat{\sigma}_z(T)\rangle$ and $W(0, T)$, respectively, for given values of the interaction parameters. Also this can be emphasized by deducing the asymptotic form for (25) in the strong-intensity regime (i.e. $|\alpha|$ is large). By means of the harmonic approximation technique [27] (see equation (A.1) in the appendix) and after straightforward calculations (25) can be expressed as

$$W(0, T) = \exp\left[-2\bar{n}\cos^2\left(\frac{T}{2\bar{n}}\right)\right]\cos\left[T\left(\bar{n} + \frac{1}{\bar{n}}\right) - \bar{n}\sin\left(\frac{T}{\bar{n}}\right)\right] \qquad (26)$$

where $\bar{n} = |\alpha|$. Expression (26) is similar to that of $\langle\hat{\sigma}_z(T)\rangle$ except $\cos(\cdot)$ in the exponent should be replaced by $\sin(\cdot)$. As a result of this fact the envelope function in (26) gives its maximum value at $T = \pi\bar{n}$, whereas that of $\langle\hat{\sigma}_z(T)\rangle$ is maximum at $T = 2\pi\bar{n}$. From these arguments and information displayed in figure 1, one can conclude that $W(0, T)$ can give similar information on the corresponding atomic inversion provided that the interaction time T is replaced by $\tau \equiv T + \pi\bar{n}$. The behaviour associated with this situation is given in figure 2. Comparison between figures 1(*a*) and 2 is instructive.

Now we turn attention to formula (20), which is related to the homodyne tomography. Firstly, it is worth recalling that the properties of the Hermite polynomial provide $\mathrm{H}_{2n+1}(0) = 0$ and $\mathrm{H}_{2n}(0) \neq 0$. These facts make the argument related to $P(\underline{0}, T)$ different from that given for $W(\underline{0}, T)$. For instance, when one of the modes is initially prepared in odd parity states (e.g. odd coherent states) and the associated transition parameter with this mode is an even number then $P(\underline{0}, T) = 0$; however, this is not the case for the corresponding W function, where $W(\underline{0}, T) \neq 0$. Also one can easily recognize that $P(\underline{0}, T) \neq 0$ for different cases, e.g. when all modes are initially in even parity (non-parity) states regardless of the values of k_j or when all modes are initially in odd parity states provided that the transition parameters are odd numbers.

Similar to the treatment given for the W function we investigate $P(0, T)$ of the single-mode case when the field is initially in coherent state, $k_1 = 1$ and $\triangle = 0$. Therefore,

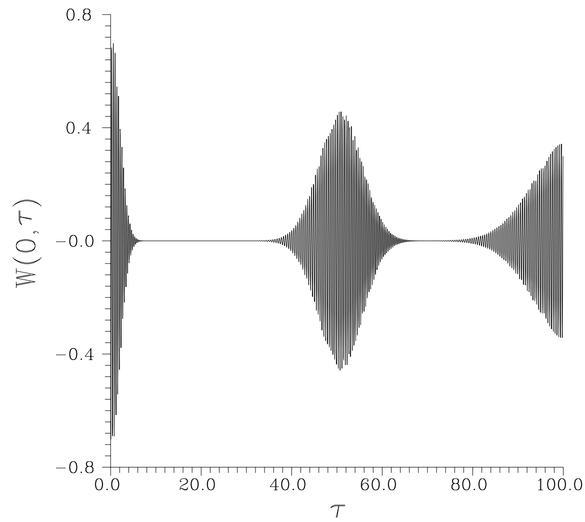

Figure 2. The evolution of the $W(0,\tau)$ against the shifted-scaled time τ for the same situation as in figure 1(b).

relation (20) can be expressed as

$$P(0,T)=\frac{1}{2}\sum_{n_1=0}^{\infty}P(n_1)\left\{\left[\frac{\mathrm{H}_{n_1}^2(0)}{2^{n_1}n_1!}+\frac{\mathrm{H}_{n_1+1}^2(0)}{2^{n_1+1}(n_1+1)!}\right]\right.$$
$$\left.+\left[\frac{\mathrm{H}_{n_1}^2(0)}{2^{n_1}n_1!}-\frac{\mathrm{H}_{n_1+1}^2(0)}{2^{n_1+1}(n_1+1)!}\right]\cos(2T\sqrt{n_1+1})\right\} \tag{27}$$

where $P(n_1)$ is the photon number distribution for coherent states. In the strong-intensity regime the asymptotic form for (27), which is corresponding to (26), is

$$P(0,T)=\frac{1}{2}\exp(-|\alpha|^2)\left\{\mathrm{I}_0(|\alpha|^2)+\mathrm{I}_1(|\alpha|^2)+\mathrm{Re}\left\{\exp\left[\mathrm{i}T\left(\bar{n}+\frac{1}{\bar{n}}\right)\right]\right.\right.$$
$$\left.\left.\times\left[\mathrm{I}_0\left(|\alpha|^2\exp\left(\mathrm{i}\frac{T}{\bar{n}}\right)\right)-\mathrm{I}_1\left(|\alpha|^2\exp\left(\mathrm{i}\frac{T}{\bar{n}}\right)\right)\right]\right\}\right\} \tag{28}$$

where $\mathrm{I}_0(\cdot)$ and $\mathrm{I}_1(\cdot)$ are the modified Bessel functions of the first kind of order 0 and 1, respectively. The derivation for (28) is given in the appendix. Expression (28) is periodic with period $2\pi\bar{n}$ where $\bar{n}$ is an integer. Additionally, $P(0,T)$ gives its maximum values around $T=m\bar{n}\pi$ whenever m is an odd integer. We have plotted (27) in figure 3 against both the scaled time T (a) and the shifted-scaled time τ (b) for the same situations as those for figures 1(b) and 2, respectively. Comparison between figures 1(b) and 3(a) as well as figures 2 and 3(b) shows that RCP in $\langle\hat{\sigma}_z(T)\rangle$ can be observed via homodyne tomography. Of course, the evolution of $\langle\hat{\sigma}_z(T)\rangle$ and $P(0,T)$ possesses different scales. This problem can be solved easily by using (28) and the information displayed in figure 3(b). For instance, in the strong-intensity regime, we can adopt the following relation,

$$\langle\hat{\sigma}_z(T)\rangle\equiv\frac{1}{P_{\max}}[P(0,\tau)-P(0,0)] \tag{29}$$

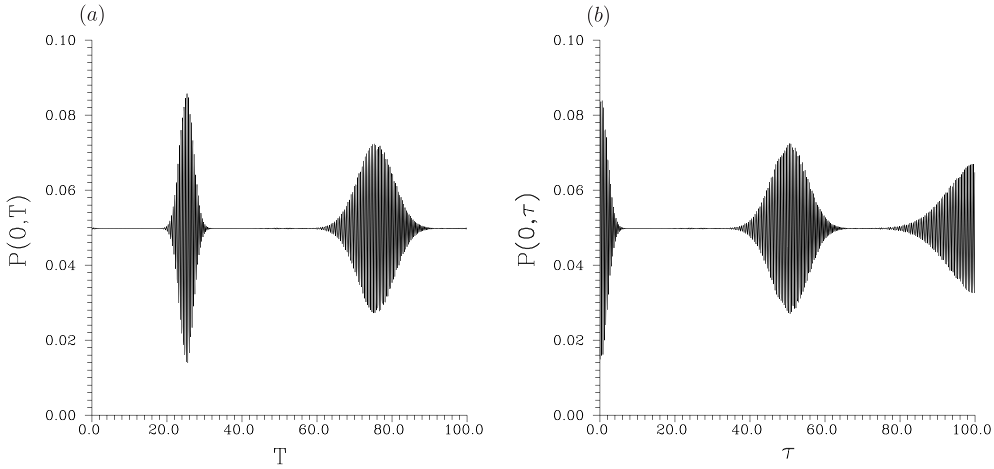

Figure 3. The probability distribution measured using homodyne tomography corresponding to the evolution of $W(0, T)$ against both the scaled time T (*a*) and shifted-scaled time τ (*b*) for the same situation as figures 1(*b*) and 2, respectively.

where

$$P(0,0) = \exp(-|\alpha|^2)\mathrm{I}_0(|\alpha|^2) \qquad P_{\max} = [\mathrm{I}_0(|\alpha|^2) + \mathrm{I}_1(|\alpha|^2)]\exp(-|\alpha|^2) \tag{30}$$

where the subscript max stands for the maximum value of $P(0, T)$. The explicit form for $P_{\max}$ can be obtained by analysing the behaviour of $P(0, T)$ around $T = \pi\bar{n}$. Finally, the origin of the prefactor (i.e. $1/P_{\max}$) in (29) can be understood as follows. $P(0, T)$ carries complete information on $\langle\hat{\sigma}_z(T)\rangle$ when it has not only similar behaviour but also the same amplitude as $\langle\hat{\sigma}_z(T)\rangle$. Thus we seek an amplification factor, say, μ (i.e. $\mu P(0, T) \equiv \langle\hat{\sigma}_z(T)\rangle$) such that

$$\mu P_{\max} \equiv |\langle\hat{\sigma}_z(T)\rangle_{\max}| = 1. \tag{31}$$

3.2. Observation and measurement

In the first part of this section we have shown that under certain conditions there is a direct relation between $\langle\hat{\sigma}_z(T)\rangle$ and $W(0, T)$ for the JCM. Such a relation indicates that the atomic inversion can be detected via techniques similar to those used for the W function. This will be discussed in the following, in particular, for the single-mode JCM.

As is well known there are different schemes proposed for measuring the W function, which are photon counting experiment [28], using a simple experiment similar to that used in cavity QED and ion traps [29, 30], and tomographic reconstruction from data obtained in homodyne measurements [31, 32]. Here we argue how these techniques can be used for measuring $\langle\hat{\sigma}_z(T)\rangle$.

(i) *Measurement of the* $\langle\hat{\sigma}_z(T)\rangle$ *using photon counting experiment.* The photon counting method is based on the fact that the single-mode W function at the origin of the phase space can be directly measured by a photodetector facing this mode [28]. Hence $\langle\hat{\sigma}_z(T)\rangle$ can be detected via this technique in the following sense (see figure 4). Generally, a mode prepared in a coherent state, which is pumped by a laser source, interacts first with the two-level excited localized atom—localized and/or very slow atoms can be prepared by means of, e.g. laser-cooling technique [33]—hence the outgoing field is superimposed by a strong

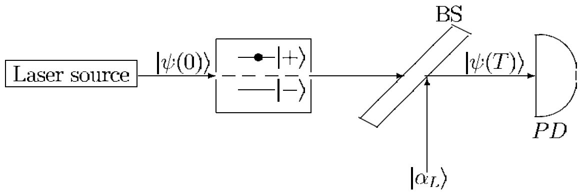

Figure 4. The proposed setup for detecting $\langle\hat{\sigma}_z(T)\rangle$ of the single-mode JCM when the field is initially prepared in the coherent light pumped by the laser source. BS and PD denote the beam splitter and photodetector, respectively. The two-level excited atom is localized in the microcavity as indicated. The initial state, detected state and local-oscillator state are denoted by $|\psi(0)\rangle$, $|\psi(T)\rangle$ and $|\alpha_L\rangle$, respectively.

local-oscillator mode ($|\alpha_L\rangle$) via a beam splitter whose transmissivity (reflectivity) is high (low). The measurement has to be performed only on one of the output ports of the beam splitter via the photodetector. It is worthwhile mentioning that the one-port measurement for the beam splitter can be performed using a conditional measurement technique in which no photons are measured in the free port, e.g. [34]. We proceed by considering a perfect photodetector, i.e. its efficiency is unity. The probability $P(n_1, T)$ of the registration of n_1 photons at time T in the detector is given by

$$P(n_1, T) = \left\langle : \frac{\hat{\Lambda}^{n_1}}{n_1!} \exp(-\hat{\Lambda}) : \right\rangle \tag{32}$$

where :: stands for the normally ordered operator, angle brackets mean expectation value (which is calculated in the framework of the Schrödinger picture) and $\hat{\Lambda}$ is the operator of the integrated flux of light onto the surface of the detector having the form

$$\hat{\Lambda} = (\sqrt{\bar{\tau}}\hat{a}^\dagger - \sqrt{1-\bar{\tau}}\alpha_L^*)(\sqrt{\bar{\tau}}\hat{a} - \sqrt{1-\bar{\tau}}\alpha_L) \tag{33}$$

where $\bar{\tau}$ is the transmissivity power of the beam splitter. The count statistics determined in the experiment is used to compute the following count generating function:

$$\begin{aligned} G(\alpha, T) &= \sum_{n_1=0}^{\infty} (-1)^{n_1} P(n_1, T) \\ &= \langle : \exp(-2\hat{\Lambda}) : \rangle. \end{aligned} \tag{34}$$

The second line in (34) is obtained by substituting (32) in the first line of this equation. It is evident that for $\triangle = 0$, $W(0, T)$ given by (23) can be expressed in the form (34), where, in this case, we have

$$P(n_1, T) = \left|C_{n_1}^{(1)}\right|^2 [\cos^2(T\sqrt{h(n_1, k_1)}) + \sin^2(T\sqrt{h(n_1, -k_1)})]. \tag{35}$$

We proceed from the definition of the W function (16) and (34) that the W function for the detected mode is proportional to $s = 1 - 1/\bar{\tau}$ ordered quasidistribution function of the mode entering the beam splitter (which is not included in figure 4) as

$$G(\alpha, T) = \frac{1}{\bar{\tau}} W\left(\sqrt{\frac{1-\bar{\tau}}{\bar{\tau}}}\alpha_L, \frac{\bar{\tau}-1}{\bar{\tau}}, T\right). \tag{36}$$

When the transmissivity of the beam splitter is near one the scanned quasidistribution is $W(0, T)$, i.e. the atomic inversion is detected. Here we assume that the interaction time in the microcavity and the detection time in the photodetector are equal since we are interested only in showing that $\langle\hat{\sigma}_z(T)\rangle$ can be measured via the photon counting technique. Nevertheless, in the realistic situation there is a time delay between the interaction and the detection processes. This problem can be solved by obtaining information on both the detector–microcavity distance and the velocity of the radiation field.

(ii) *Measurement of the* $\langle\hat{\sigma}_z(T)\rangle$ *using cavity QED*. In this technique, probes of the two-level atoms interact dispersively with the field under consideration—the source of the field in this case is the JCM by which we would like to measure $\langle\hat{\sigma}_z(T)\rangle$—causing a phase shift to the atomic wavefunction, which is proportional to the photon number. This phase shift can be revealed by the Ramsey atomic interferometer [35]. Under certain conditions the final state of the atom measures the field parity at the phase-space origin, i.e. $W(0, T)$. The experimental setup related to this proposal is given in [29] (see figure 1 in [29]) but with additional arrangements. For instance, in this setup we have to block the microwave generator, which causes displacement of the field under consideration since the interest is focused only on the behaviour of the W function at the phase-space origin.

(iii) *Measurement of the* $\langle\hat{\sigma}_z(T)\rangle$ *using homodyne tomography arrangement*. Generally single-mode homodyne tomography is based on the set of distributions $pr(q_1, \theta_1)$ measured by homodyne detection, i.e. the field to be measured beats with the local oscillator in a homodyne arrangement [36, 37]. Once $pr(q_1, \theta_1)$ is obtained the W function can be reconstructed via the inverse Radon transformation. At the phase-space origin this relation reads

$$W(0, T) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}\zeta \int_{-\infty}^{\infty} \mathrm{d}\eta|\eta|pr(\zeta, T)\exp(\mathrm{i}\eta\zeta) \tag{37}$$

where $pr(\zeta, T) = \langle\zeta|\hat{\rho}(T)|\zeta\rangle$. The value of the relative phase between the local oscillator and the signal field, which is assumed to be the field outgoing from the JCM microcavity, is zero. This can be arranged by moving a mirror on a piezoelectric translator [38]. It is worthwhile mentioning that $pr(\zeta, T)$ has been measured via this technique, e.g. in [38]. Also investigation for the random-phase states using the homodyne tomography technique is given in [39].

Finally, we conclude that the techniques (i) and (ii) can lead to a direct measurement of $\langle\hat{\sigma}_z(T)\rangle$. Also, they do not involve the inversion algorithm and hence they should be much less sensitive to experimental errors than the tomographic technique. Furthermore, they can be, in principle, applied to the atomic inversion of the multimode JCM where attention has to be focused on the measurement of the $\langle\hat{\sigma}_z(T)\rangle$ in its entangled form.

4. Conclusions and remarks

In this paper we have discussed the relation between both the evolution of the atomic inversion and the corresponding W function for the multimode multiphoton JCM. We have shown that under certain conditions there is a direct relation between these two quantities, which is valid for resonance and off-resonance cases. Such a relation suggests that the nonclassical effects stored in the radiation field can be noticed through the behaviour of $\langle\hat{\sigma}_z(T)\rangle$. Furthermore, based on this relation, we have discussed the possibility of detecting $\langle\hat{\sigma}_z(T)\rangle$ for the single-mode case using techniques similar to those applied to the W function.

The results given in this paper are valid for any JCM Hamiltonian provided that it has been deduced in the framework of rotating wave approximation. More illustratively, if in (1) the operators $\prod_{j=1}^{N}\hat{a}_j^{k_j}$ are replaced by $\hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{a}_3^{k_3}\hat{a}_4^{\dagger k_4}\cdots$, i.e. some of the annihilation operators are replaced by creation ones, relation (17) will not be affected owing to the fact

$$\langle n-k|\exp(\mathrm{i}\pi\hat{a}^\dagger\hat{a})|n-k\rangle \equiv \langle n+k|\exp(\mathrm{i}\pi\hat{a}^\dagger\hat{a})|n+k\rangle = (-1)^{n+k}. \tag{38}$$

Moreover, the results are independent of the type of initial atomic state (i.e. if the atom is in the excited state, ground state or atomic superposition state). Basically expression (17) depends on the Fock state basis of the dynamical wavefunction, e.g. for the single-mode case it depends on $|n\rangle$, $|n+k\rangle$ and $|n-k\rangle$. Consequently, such a relation still exists when the Jaynes–Cummings Hamiltonian includes Kerr nonlinearity [40], Stark effect [41], intensity dependence [42] and atomic motion [43]. Furthermore, among all quasiprobability distribution functions such a relation, i.e. (17), exists only for the W function. For instance, the evolution of the Q function at the phase-space origin for the single-mode JCM with initial coherent light and $\triangle = 0$ is given by

$$Q(0,T) = \exp(-2|\alpha|^2)\cos^2(T\sqrt{k_1!}). \tag{39}$$

It is obvious that in the strong-intensity regime, $Q(0,T) \to 0$.

Finally, as is well known, the W function is a global quantity which characterizes the full quantum state. Additionally, dealing with the W function at an isolated single point leads to difficulty in finding a proper normalization. Nevertheless, throughout the paper we have focused the attention on the behaviour of WOP and its relation with the behaviour of $\langle\hat{\sigma}_z(T)\rangle$. Therefore the normalization has no effect on the dynamical behaviour of the system.

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Appendix

In this appendix we derive the asymptotic form (28) for $P(0,T)$. It is worth remembering that in the strong-intensity regime (i.e. $|\alpha| \gg 1$) the argument of $\cos(\cdot)$ in (27) can be expressed as [27]:

$$\sqrt{n+1} = \sqrt{\langle\hat{n}\rangle + n + 1 - \langle\hat{n}\rangle} \simeq \frac{1}{2}\left(\bar{n} + \frac{1}{\bar{n}} + \frac{n}{\bar{n}}\right) \tag{A.1}$$

where $\bar{n} = \sqrt{\langle\hat{n}(0)\rangle}$.

Now we show how the different summations in (27) can be evaluated.

$$\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{(n!)^2 2^n}\mathrm{H}_n^2(0) = \frac{1}{2\pi}\int_0^{2\pi} \sum_{n=0}^{\infty} \frac{\left(\frac{\alpha}{\sqrt{2}}\right)^n}{n!}\mathrm{H}_n(0) \sum_{m=0}^{\infty} \frac{\left(\frac{\alpha^*}{\sqrt{2}}\right)^m}{m!}\mathrm{H}_m(0)\,\mathrm{d}\phi \tag{A.2}$$

where $\alpha = |\alpha|\exp(\mathrm{i}\phi)$. By means of the generating function of the Hermite polynomial the summations on the right-hand side of (A.2) can be carried out as

$$\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{(n!)^2 2^n}\mathrm{H}_n^2(0) = \frac{1}{2\pi}\int_0^{2\pi} \exp[-|\alpha|^2\cos(2\phi)]\,\mathrm{d}\phi = \mathrm{I}_0(|\alpha|^2) \tag{A.3}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind of order zero. The second summation we would like to evaluate is

$$\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!(n+1)!2^{n+1}}\mathrm{H}_{n+1}^2(0) = \sum_{m=0}^{\infty} \frac{|\alpha|^{2(m-1)}}{(m-1)!m!2^m}\mathrm{H}_m^2(0) \tag{A.4}$$

where the factorial $-1!$ is ∞. Summation (A.4) can be reformulated as

$$\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n!(n+1)!2^{n+1}}\mathrm{H}_{n+1}^{2}(0)=\frac{\mathrm{d}}{\mathrm{d}|\alpha|^{2}}\sum_{m=0}^{\infty}\frac{|\alpha|^{2m}}{(m!)^{2}2^{m}}\mathrm{H}_{m}^{2}(0). \tag{A.5}$$

Using (A.3), the right-hand side of (A.5) gives

$$\frac{\mathrm{d}}{\mathrm{d}|\alpha|^{2}}\mathrm{I}_{0}(|\alpha|^{2})=\mathrm{I}_{1}(|\alpha|^{2}). \tag{A.6}$$

The terms including $\cos(\cdot)$ in the strong-intensity regime can be written as

$$\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{(n!)^{2}2^{n}}\mathrm{H}_{n}^{2}(0)\cos\left(T\left(\bar{n}+\frac{1}{\bar{n}}+\frac{n}{\bar{n}}\right)\right)=\mathrm{Re}\left\{\exp\left[\mathrm{i}T\left(\bar{n}+\frac{1}{\bar{n}}\right)\right]\right.$$
$$\left.\times\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{(n!)^{2}2^{n}}\mathrm{H}_{n}^{2}(0)\exp\left(\mathrm{i}T\frac{n}{\bar{n}}\right)\right\}. \tag{A.7}$$

The summation on the right-hand side of (A.7) can be evaluated using procedures as those given above leading to

$$\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{(n!)^{2}2^{n}}\mathrm{H}_{n}^{2}(0)\cos\left(T\left(\bar{n}+\frac{1}{\bar{n}}+\frac{n}{\bar{n}}\right)\right)=\mathrm{Re}\left\{\exp\left[\mathrm{i}T\left(\bar{n}+\frac{1}{\bar{n}}\right)\right]\mathrm{I}_{0}\left(|\alpha|^{2}\exp\left(\mathrm{i}\frac{T}{\bar{n}}\right)\right)\right\}. \tag{A.8}$$

Similarly the last summation in (27) can be performed.

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